Aseismic design of reinforced concrete frames using inelastic response spectra

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ABSTRACT : This paper aims at generation of inelastic response spectra of viscously damped single degree of freedom systems. These spectra consist of yield displacement spectra, constant strength spectra, constant ductility spectra, reduction factor spectra, inelastic acceleration spectra and inelastic displacement spectra. Three hysteresis models: elasto-plastic, bilinear and stiffness degrading models are used to generate the above spectra for various ground motions. An attempt is also made to evaluate the effectiveness of the inelastic response spectra design approach. A six storey frame so designed is subjected to the Koyna, El Centro and an artificial earthquakes. The ductility requirements are computed by a non-linear time history analysis method. It is concluded that the inelastic response spectra design approach is conservative and satisfactory.

1 INTRODUCTION

During the past several years our ability to analyze structures for the effects of earthquake ground motions has increased considerably. Still, there is no reliable technique for the design of structures. It is well recognised that structures required to resist strong ground motions should be capable of withstanding substantial inelastic deformations. The elastic analysis-design approach as specified in most building codes is not satisfactory because it fails to predict the location and extent of plastification in a structure. Hence, there is a need of a suitable inelastic analysis-design approach.

A structure can be designed based on inelastic time-history analysis for the expected ground motion. This is an iterative process of design and analysis which is very cumbersome in application. Besides cost in time and money, the proper ground motion to be used as input cannot be determined, and the results are not easily interpreted. Hence this approach has very limited applications.

In the preliminary design of structure, one is primarily concerned with the maximum response rather than the precise details of the response. Thus, it would be helpful to develop design guidelines which indicate how these peak response parameters vary with the dynamic, mechanical and damping characteristics of a structure. Such guidelines can be formulated for single degree of freedom systems in the form of inelastic response spectra. The concept of inelastic response spectra was first introduced by Veletsos et.al.(1965). Later, Newmark et.al.(1969, 1980), Murakami and Penzien (1977), Lai and Biggs (1980), and Briseghella et.al.(1982) carried out statistical study on the response spectra for different hysteresis models and ground motions. They proposed different alternatives to construct the inelastic response spectra. Chandrasekaran and Saini (1967) proposed reduction factors for the El Centro, Taft and Koyna earthquakes for the design of non-linear systems.

This paper aims at generation of inelastic response spectra of viscously damped single degree of freedom systems. These spectra consist of yield displacement spectra, constant strength spectra, constant ductility spectra, reduction factor spectra, inelastic acceleration spectra and inelastic yield displacement spectra. Three hysteresis models: elasto-plastic, bilinear and stiffness degrading models are used to generate the above spectra for the El Centro, Koyna and an artificial earthquake. In this paper an attempt is also made to evaluate the effectiveness of the inelastic response spectrum design approach.

2 INELASTIC RESPONSE SPECTRA

A simple means of representing structural response to a given earthquake motion is through a tripartite logarithmic response spectra. The response of a non-linear system can be represented by means of an Inelastic Yield Displacement Spectra(IYDS) or Inelastic Acceleration Spectra(IAS). In the case of former the yield deformation x_y necessary to limit the maximum deformation of the system to a specified multiple of the yield deformation itself $x_{max} = \mu x_y$ is plotted on the displacement axis. The spectral acceleration $\omega^2 x_y$ multiplied by the mass gives the yield resistance

$$R_y = m\omega^2 x_y = k x_y \tag{1}$$

which in the case of elasto-plastic system is the maximum force in the spring. For bilinear and stiffness degrading systems with strain hardening slope k_s, the maximum force in the spring is given by

$$R_{max} = m\omega^2 x_y (1+k_s(\mu-1)) \tag{2}$$

where, m = mass of single degree of freedom system, k = elastic spring stiffness, ω = frequency of vibration in radians per second

$= \sqrt{k/m}$

μ = displacement ductility

Sometimes it is desirable to deal directly with the maximum forces. It is possible to plot R_{max}/m on the acceleration axis to obtain inelastic acceleration spectra(IAS).

2.1 Equation of motion

The equation of motion of a single degree of freedom nonlinear system is given as follows:

$$m\ddot{x}(t) + c\dot{x}(t) + R(t) = p(t) = -m\ddot{y}(t) \tag{3}$$

where c is viscous damping coefficient, R is restoring force, x is displacement of the mass relative to the base, y is ground displacement and t is time. The dots represent differentiation with respect to time.

The change in stiffness at a given time step is obtained from the load deformation of hysteresis curve of the system based on the deformation computed during the time step. The viscous damping is assumed proportional to the mass of the system. This equation is solved using the step-by-step linear acceleration method. The strength factor β is defined as the ratio of yield resistance and the maximum inertial force for a given earthquake motion, that is,

$$\beta = \frac{R_y}{m\,\ddot{y}_{max}} \tag{4}$$

where R_y = yield resistance

$= k\,x_y$ (5)

Alternatively, structures with seismic resistance coefficient α_y can be defined as

$$R_y = \alpha_y W = \alpha_y mg \tag{6}$$

then $\beta = \dfrac{R_y}{m\,\ddot{y}_{max}} = \dfrac{\alpha_y mg}{m\,\ddot{y}_{max}}$

$$= \frac{\alpha_y}{\ddot{y}_{max}/g} \tag{7}$$

In this form the strength factor β reveals strength of the system as a fraction of its weight relative to the peak ground acceleration expressed as a fraction of gravity.

The equation of motion can be solved for elasto-plastic model, bilinear model or stiffness degrading model. The stiffness degrading model is decomposed into dual component model to separate the elastic component due to strain hardening for ease of inelastic state determination. The non-linear component is shown in Fig. 1 and its details are given by Pal et.al. (1987).

2.2 Ground motions

In this study three ground motions were used: N-S component of the El Centro earthquake of 1940, longitudinal component of the Koyna earthquake of December

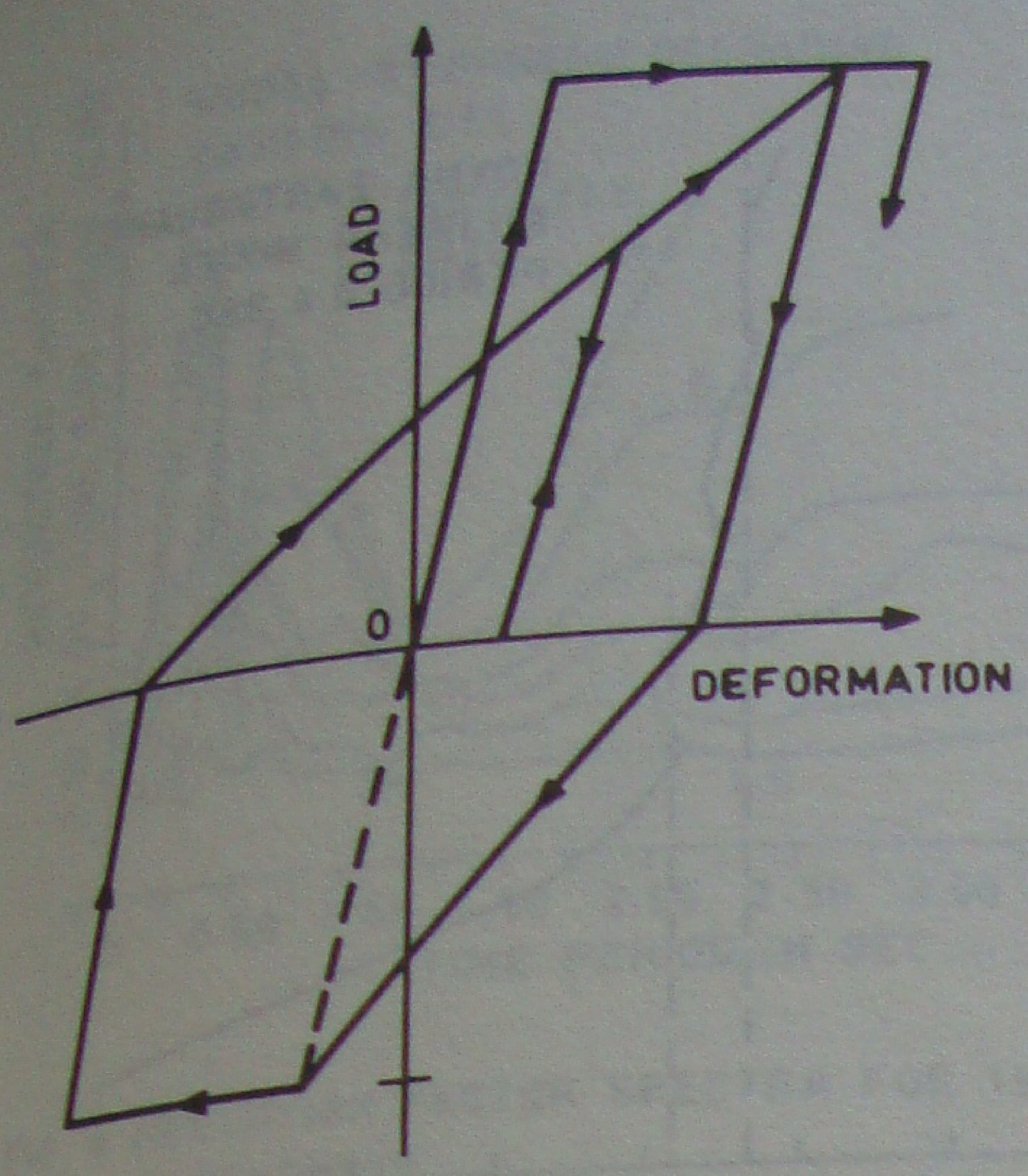

FIG. 1 STIFFNESS DEGRADING MODEL

1967 and artificial earthquake type B (five numbers). The artificial earthquakes were generated using the nonstationary Gaussian shot noise modelling. Each type B earthquake was of 30 seconds duration and the peak ground accelerations were 0.35g, 0.34g, 0.24g, 0.31g and 0.245g.

3 GENERATION OF SPECTRA

A computer program, inelastic response spectra (IRS), was written to generate the various response spectra. These spectra are automatically plotted using the CALCOMP plotter and the DEC 2050 computer. The equation of motion was integrated using a time step of 0.01 second. The inelastic spectra were generated using 24 values of the strength factor β ranging from 0.02 to 0.10 at 0.02 intervals, upto 1.5 at 0.1 intervals, and upto 4 at 0.5 intervals. The displacement ductility ratios were 1,1.5,2,2.5,3,4,5, 6,7 and 8. The curve corresponding to a ductility of 1 means elastic spectra. The response was computed for 44 values of the time periods ranging from 0.05 second to 1 second at 0.05 sec intervals, upto 3 sec at 0.1 sec intervals, and then upto 4 sec at 0.25 sec intervals. The results were obtained for all the three hysteresis models and for each of the three ground motions. The detailed results can be seen in Jain (1985). Here only typical results for the Koyna earthquake are presented.

3.1 Yield displacement spectra

For a given strength factor, it is of interest to know the variation of yield displacement with time period. This information is independent of the hysteresis model and depends on the mechanical properties of the single degree of freedom system only. This spectra gives physical idea of the order of displacement in the inelastic analysis. Figure 2 shows yield displacement spectra for the Koyna earthquake.

3.2 Constant strength spectra

In design one is usually interested in controlling the displacement ductility μ. A plot of ductility ratio μ as a function of time period for constant values of the strength factor β and damping ratio ξ is shown in Fig.3. The displacement ductility ratios tend to decrease with increasing β values and with increase in time period for all earthquake records and hysteresis models. Sometimes, lines of constant β values cross over one another. This implies that weaker systems sometimes require less ductility than stronger ones. These cross overs correspond to a sharp drop with negative slope in the β values versus displacement ductility curves as shown in Fig.4. This tendency is a result of several factors which are difficult to isolate. A similar trend was observed by Mahin and Lin (1983). In the interpolation procedure used in the computer program IRS, if more than one value of strength factor is encountered for a particular time period and desired ductility, the largest value of strength factor is selected so as to be on conservative side in the design.

The constant strength spectra can be used to determine the maximum ductility for a particular system, time period and yield strength. The maximum displacement can then be determined by multiplying this value of maximum ductility by the value of yield displacement corresponding to that time period and yield strength from the plots such as shown in Fig.2.

3.3 Constant ductility spectra

Figure 5 shows a constant ductility spectra. It is observed that the strength factor generally decreases with increase in ductility and with increase in time period. Constant ductility curves do not

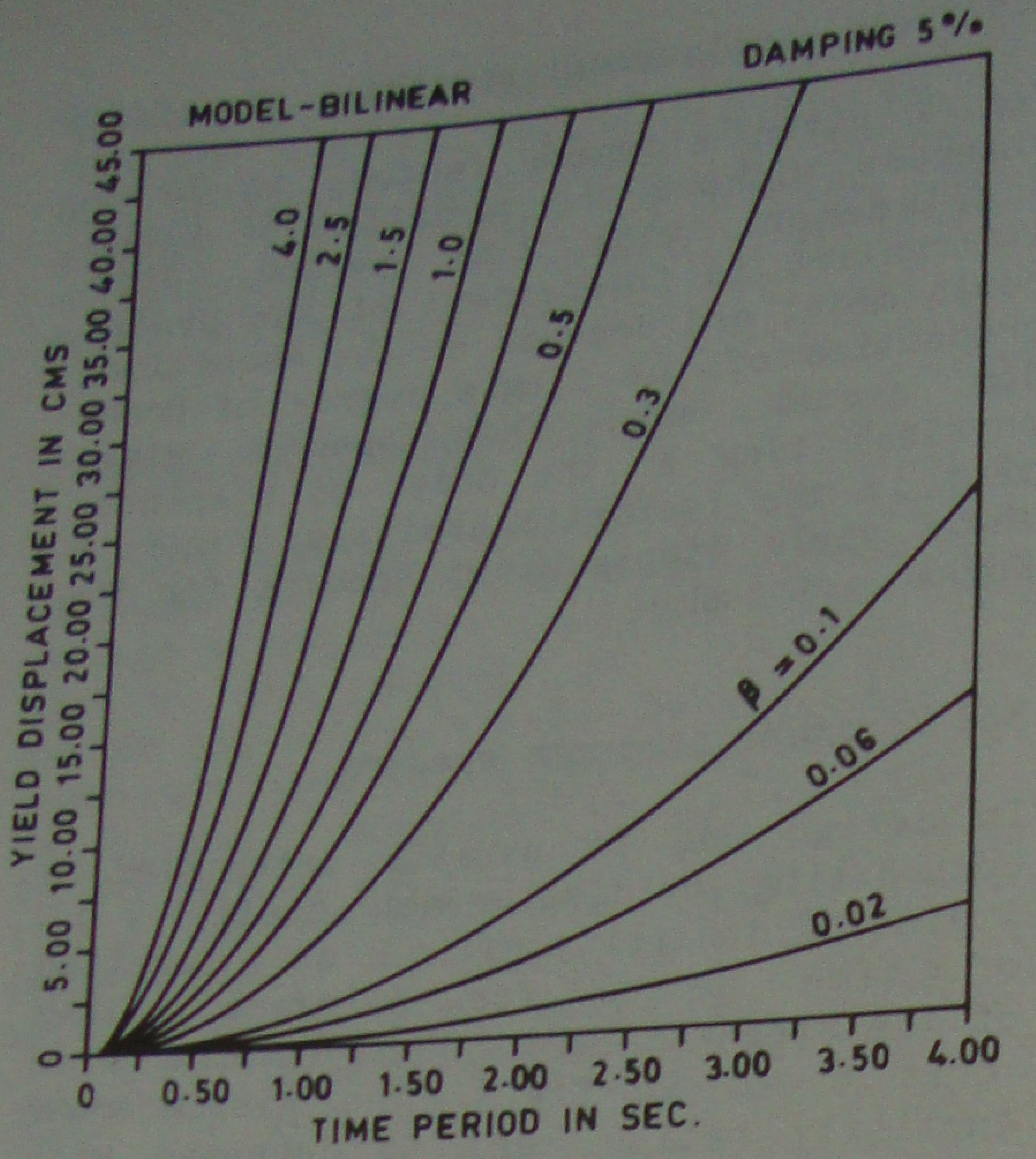

FIG. 2 YIELD DISPLACEMENT VARIATION FOR 1967 KOYNA EARTHQUAKE

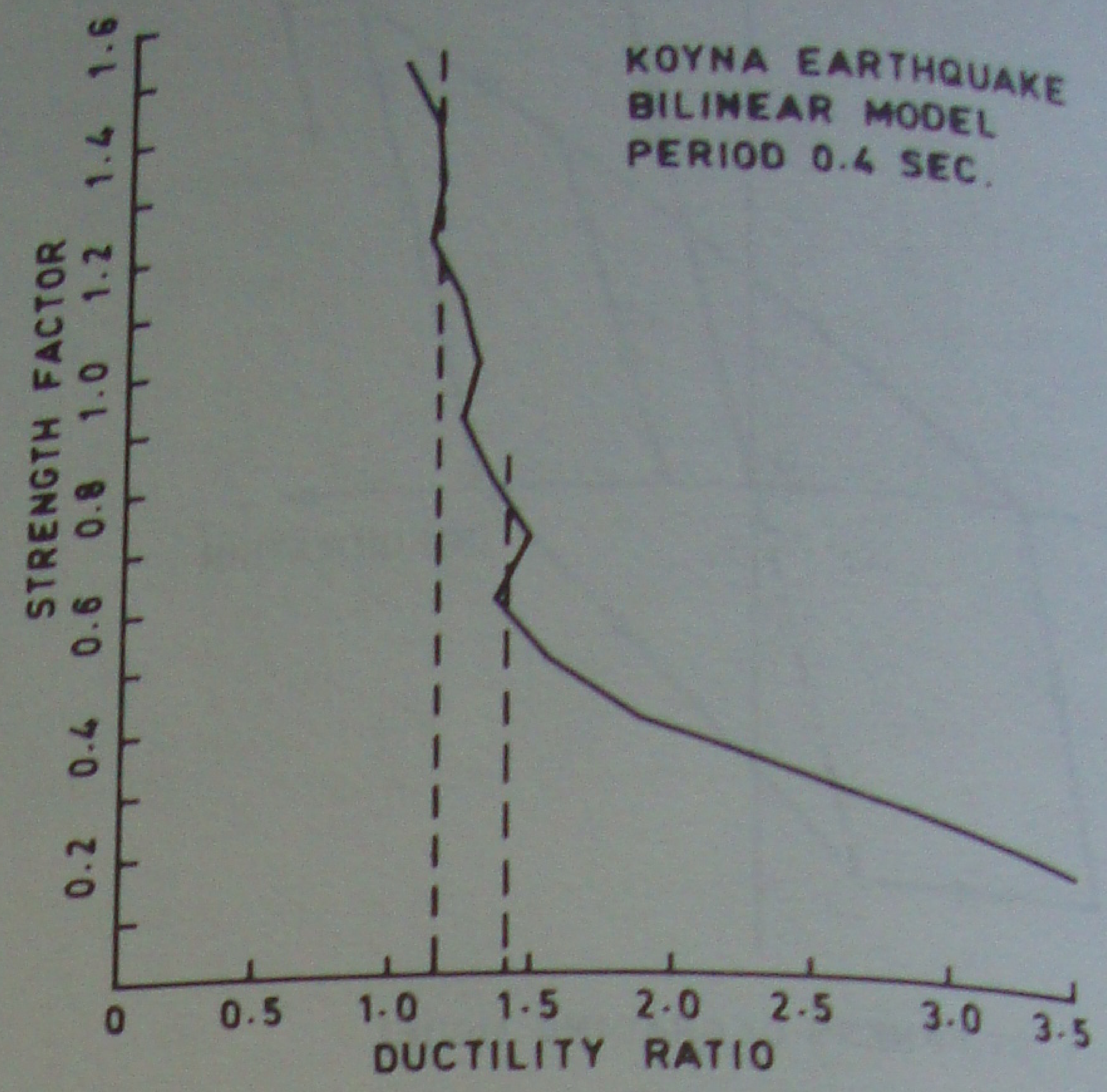

FIG. 4 VARIATION OF STRENGTH FACTOR WITH DUCTILITY

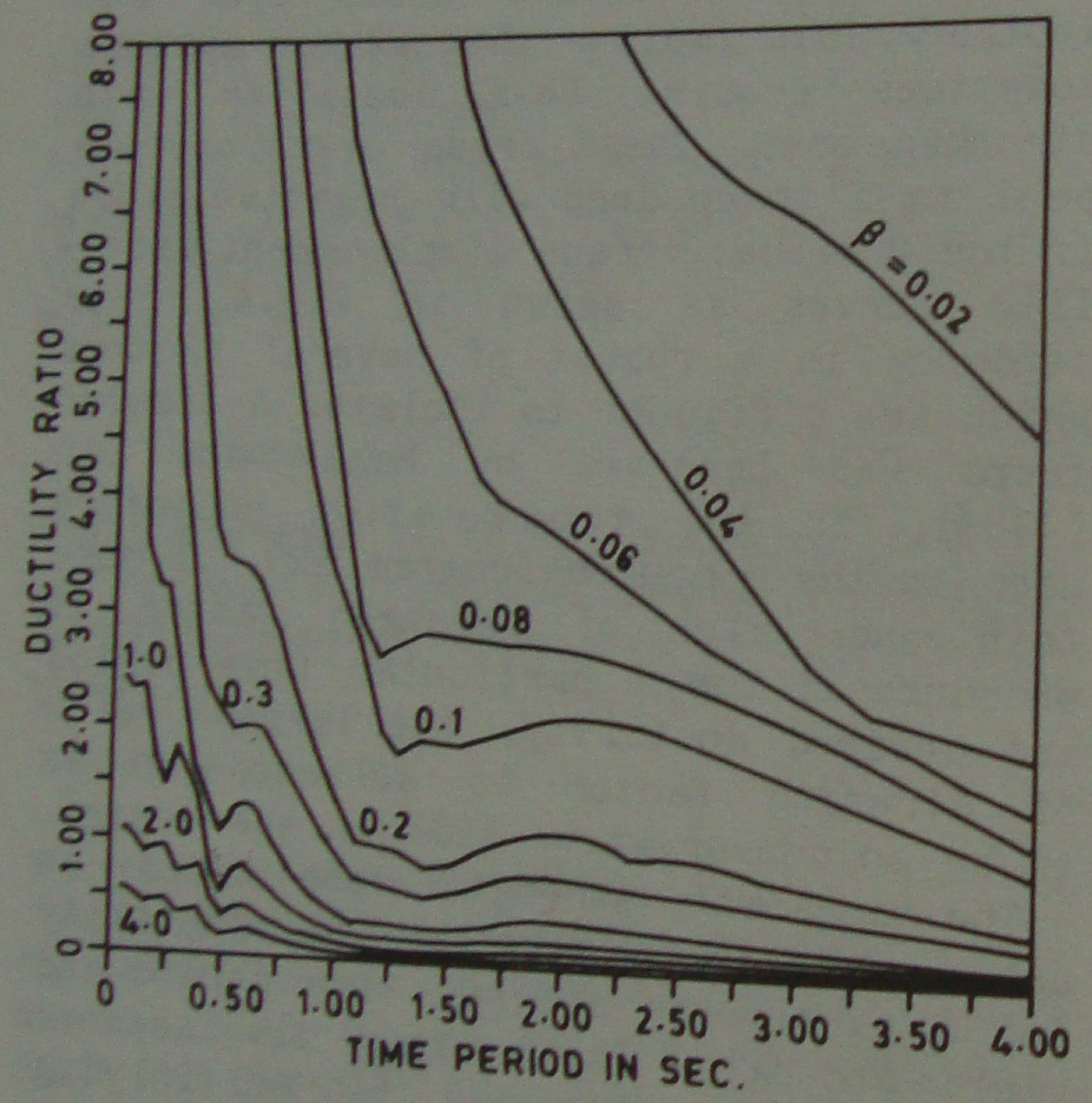

FIG. 3 CONSTANT STRENGTH RESPONSE SPECTRA FOR 1967 KOYNA EARTHQUAKE

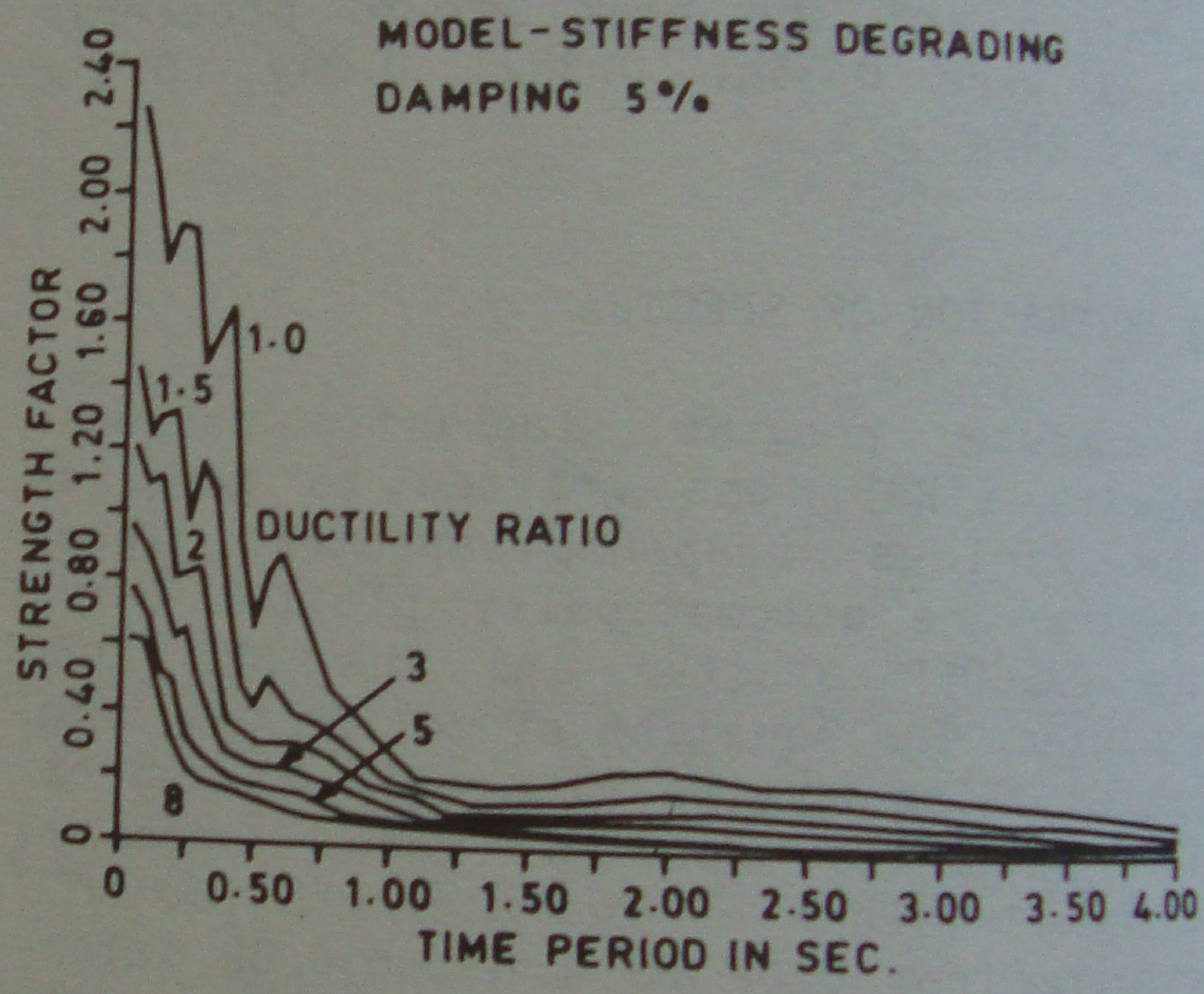

FIG. 5 CONSTANT DUCTILITY RESPONSE SPECTRA FOR 1967 KOYNA EARTHQUAKE

cross over each other. The effect of ductility ratios on strength factor is more pronounced in the lower range of desired ductility ratios. For ductility ratios of 5 and above the strength factors are nearly the same in the higher period range above 1.5 sec.

3.4 Reduction factor spectra

Reduction factor spectra for the Koyna earthquake is shown in Fig.6. Reduction factors increase with increase in ductility ratios. For ductility values of upto about 3, the reduction factors are almost constant in the time period range of 1 to 2.5 sec, whereas for higher ductility ratio the reduction factors vary significantly in different period ranges.

Finally, inelastic acceleration spectra is plotted in Fig.7 using the reduction

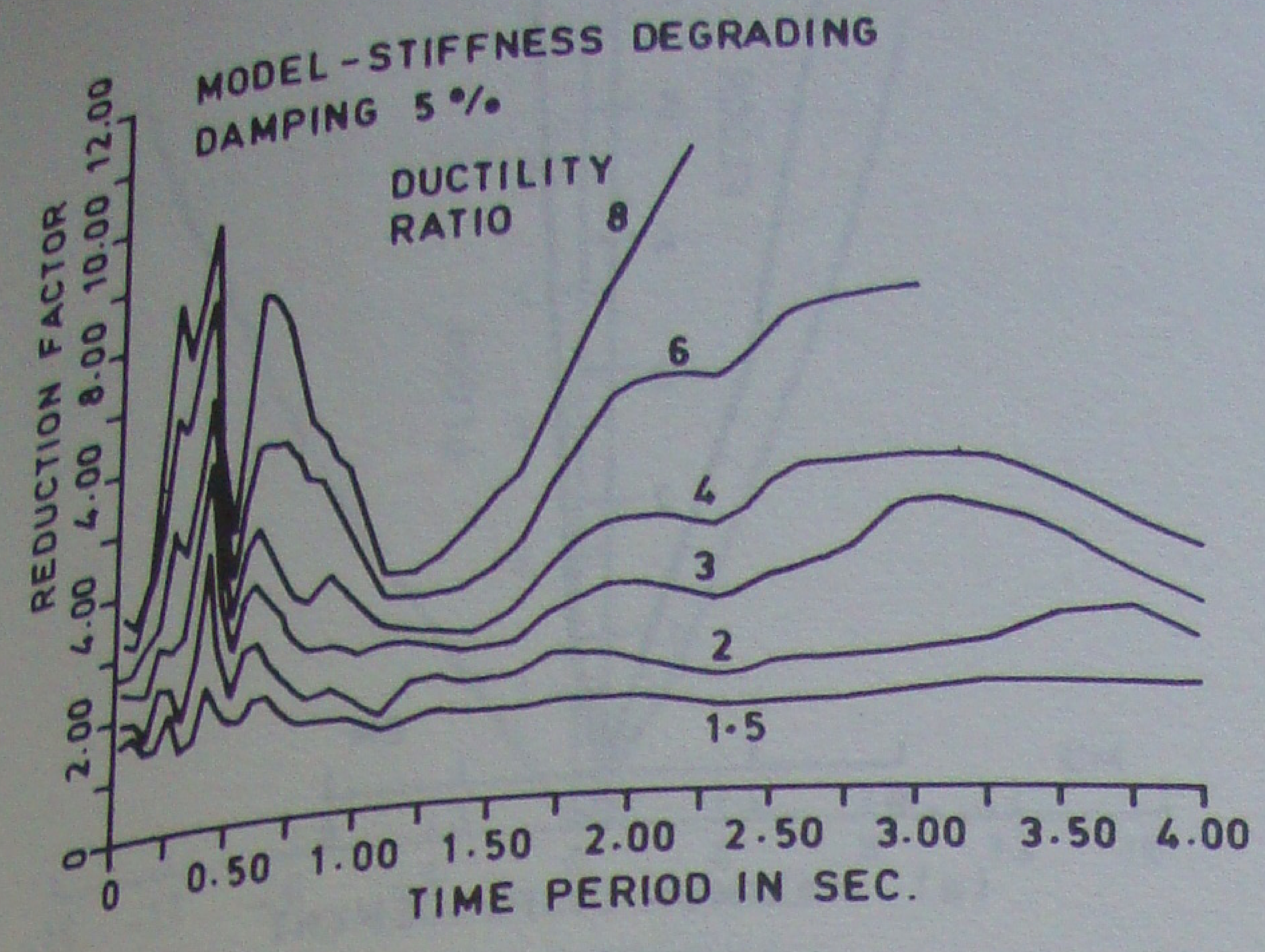

FIG. 6 REDUCTION FACTOR SPECTRA FOR 1967 KOYNA EARTHQUAKE

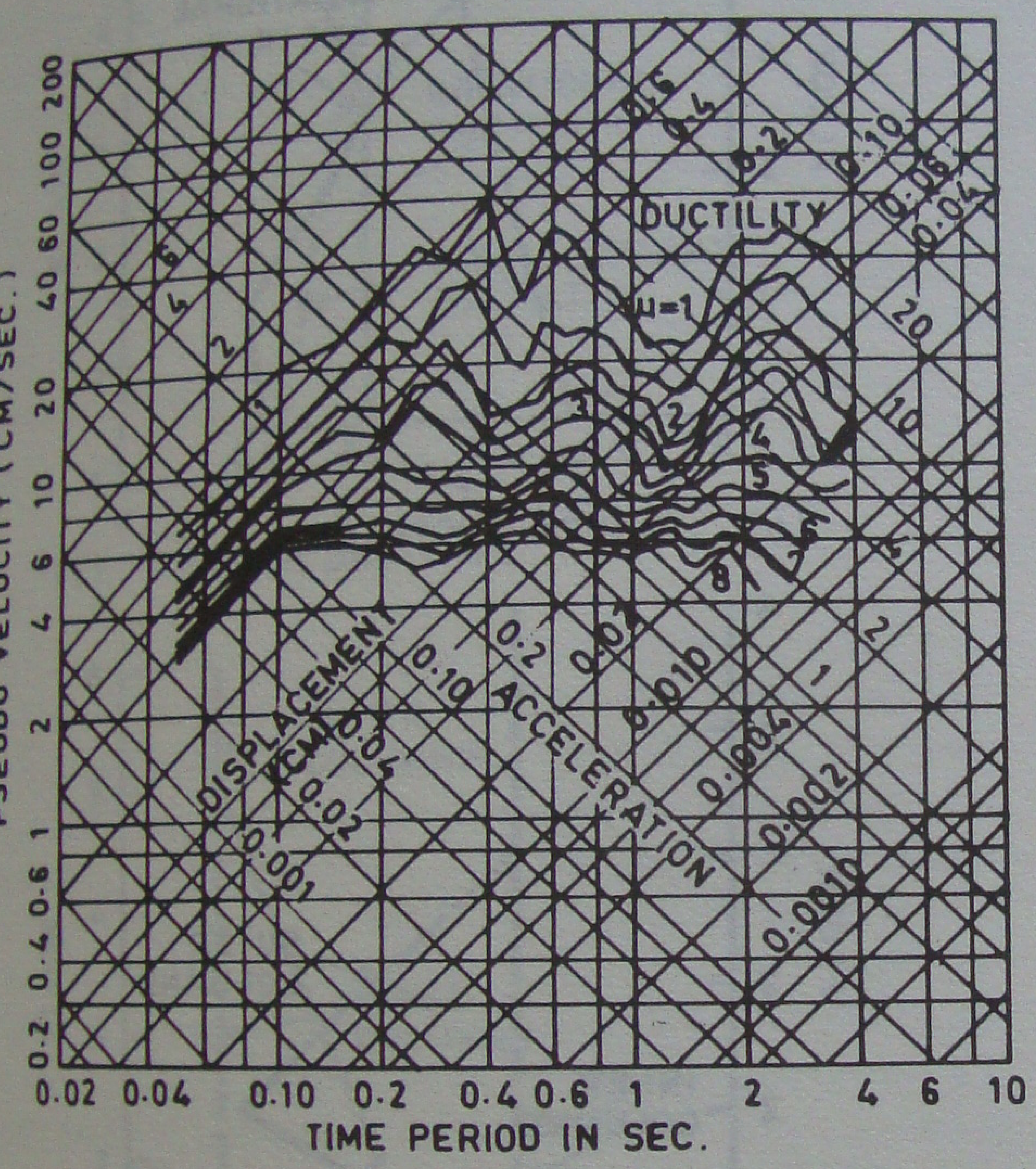

FIG. 7 INELASTIC RESPONSE SPECTRA FOR 1967 KOYNA EARTHQUAKE

factors so obtained on the acceleration axis of the tripartite log paper. The quantities on the displacement and velocity axes are meaningless. This spectra can be directly used in the design of structures.

4 ASEISMIC DESIGN OF A FRAME

A two bay, six storey moment resisting frame was selected for this study. The dead and live loads at service were 27.5 kN/m and 20 kN/m respectively. The total effective weight W was 2900 kN and the base shear was 170 kN. The frame was designed for limit state of collapse using M15 concrete (28 days cube strength = 15 MPa) and Fe 415 grade steel (yield stress = 415 MPa). The size of girders was 30 cm by 50 cm and that of columns was 30 cm by 60 cm. The frame members were designed so that sum of column moments is greater than the sum of girder moments at a joint. The sagging moment capacity of beams was taken at-least 1/2 of the hogging moment capacity to ensure enough strength against reversal of stresses.

It was noticed that the design of floor beams was governed by earthquake loading and that of the roof beam by gravity loading. The outer columns in the lower four storeys were governed by the earthquake loading. The rest of the columns were governed by the condition that sum of the column moments should be greater than sum of the girder moments at a joint. Thus, it is obvious that the inelastic response spectra will affect the design of only those members that are governed by the earthquake loading. For other members no economy may be achieved by reducing level of the earthquake forces. However, the governing conditions for different members may change depending upon the magnitude of gravity and earthquake loading.

For a total weight of 2900 kN of a frame, and a base shear of 170 kN the effective seismic coefficient is 0.06. The fundamental time period of the structure was 1.32 sec. For this time period, the acceleration coefficient of 0.06 corresponds to a displacement ductility of 2.5 on the Koyna inelastic response spectra. Similarly, the corresponding displacement ductilities on the El Centro and the artificial earthquake inelastic response spectra are 4.8 and 5.5, respectively. It means that a frame which is designed for a base shear of 170 kN, may be considered to have been designed for a ductility of 2.5, 4.8 and 5.5 for the Koyna, El Centro and the artificial earthquakes by using the respective inelastic response spectra.

Inelastic time history analysis of the example frame was carried out in order to evaluate the effectiveness of the inelastic response spectra method. Takada's stiffness degrading model was used to simulate the hysteresis behaviour of reinforced concrete girders and an elasto-plastic model with axial force-moment interaction was assumed for columns (Kanaan and Powell 1973). The frame was subjected to each of the three ground motions and the results are shown in Figs. 8,9 and 10 for the Koyna, El Centro and the artificial earthquakes, respectively. Only three response parameters are shown : maximum horizontal displacements of floors, rotational ductilities of girders and columns. The base shear-roof displacement curve of the frame was also plotted under monotonically increasing lateral loads. The yield displacement for the roof level was 5.6cm. The displacement ductility requirements were 1.1, 2.3 and 1.6 under the Koyna, El Centro and the artificial earthquakes against the design displacement ductilities of 2.5, 4.8 and 5.5. The displacement ductility requirements of the frame are also shown in Figs. 8,9 and 10. The ductility requirement from the time history analysis comes out to be much less than the ductility based on the inelastic response spectra. Thus, it can be concluded that the design based on the inelastic response spectra is conservative as compared to the results obtained from the time history analysis.

5 CONCLUSIONS

Based on the results presented in this paper, the following significant conclusions can be made:

1 The inelastic time history analysis indicated that a frame requires lesser displacement ductility than that for which it has been designed using the inelastic response spectra. Hence, the latter design approach is conservative and satisfactory. It gives the designer some control over the amount of yielding.

2 By using the proposed reduction factors, the design seismic loads for strong motion earthquakes can be reduced below those required for elastic analysis by a factor of about 1.1 to 15 depending upon the time period of the structure and the desired displacement ductility.

3 The computer program developed in this study can be used to generate constant strength spectra, constant ductility spectra, reduction factor spectra, inelastic acceleration spectra and inelastic yield displacement spectra for any hysteresis model and ground motion.

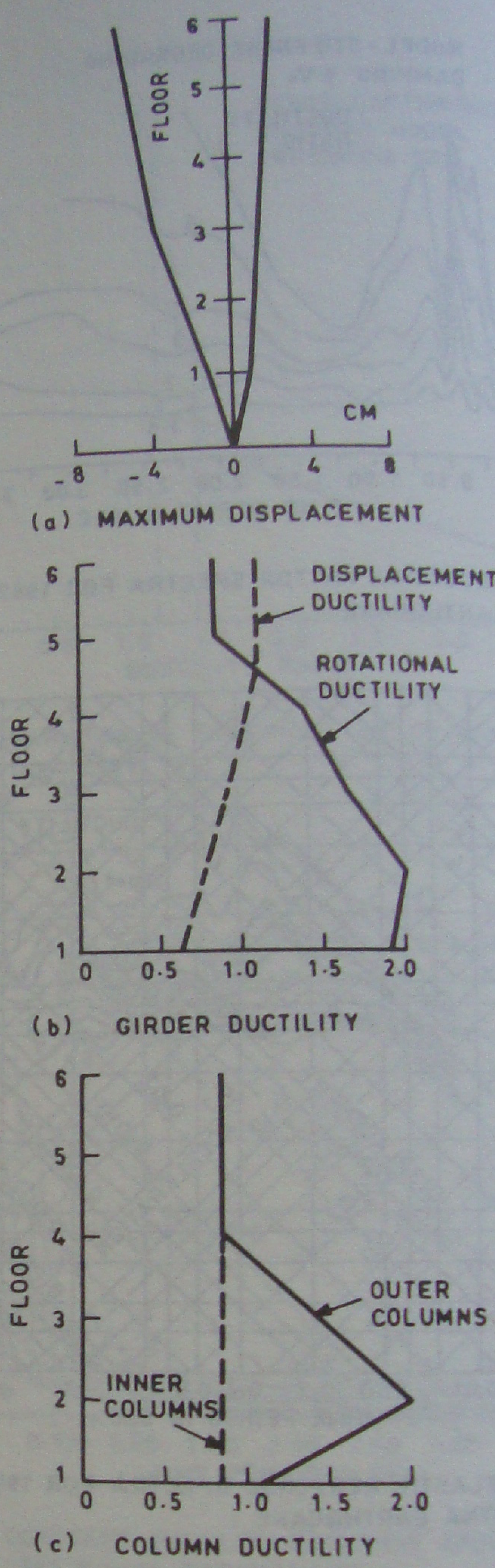

FIG. 8 RESPONSE OF FRAMES UNDER KOYNA EARTHQUAKE

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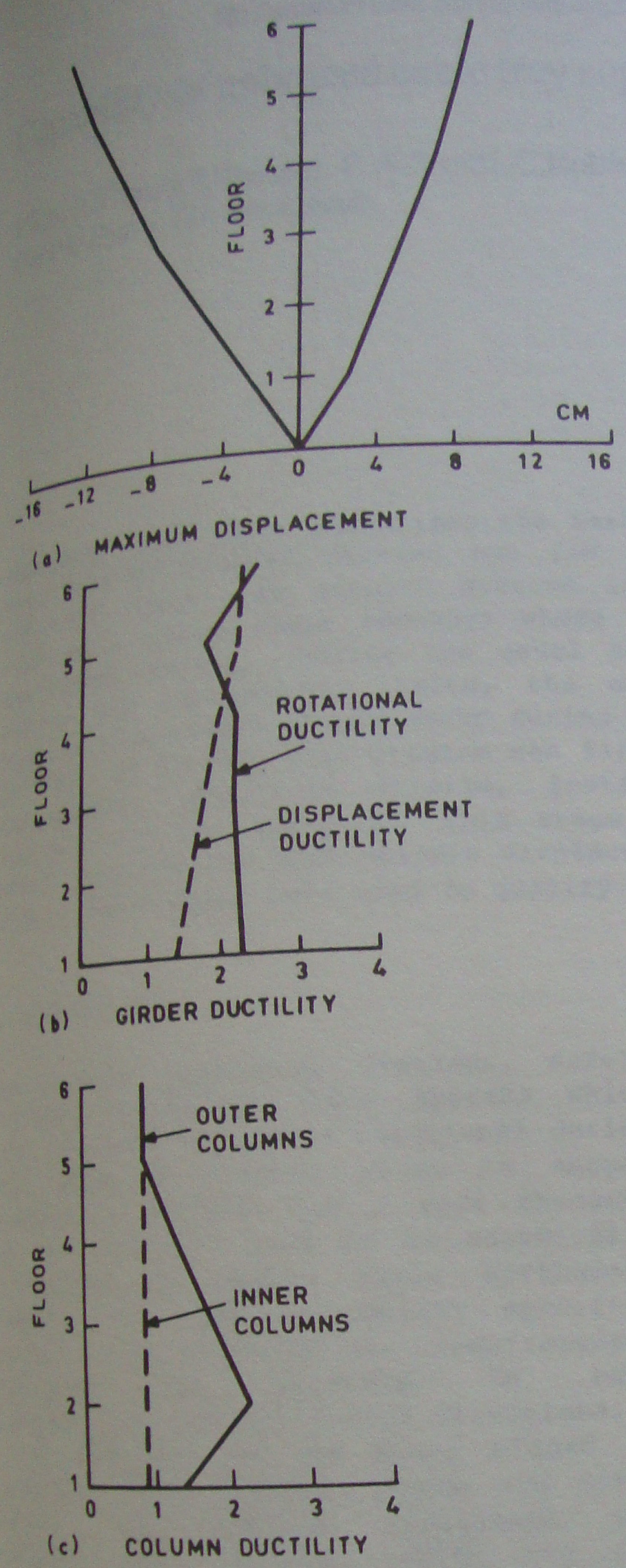

FIG. 9 RESPONSE OF FRAMES UNDER EL CENTRO EARTHQUAKE

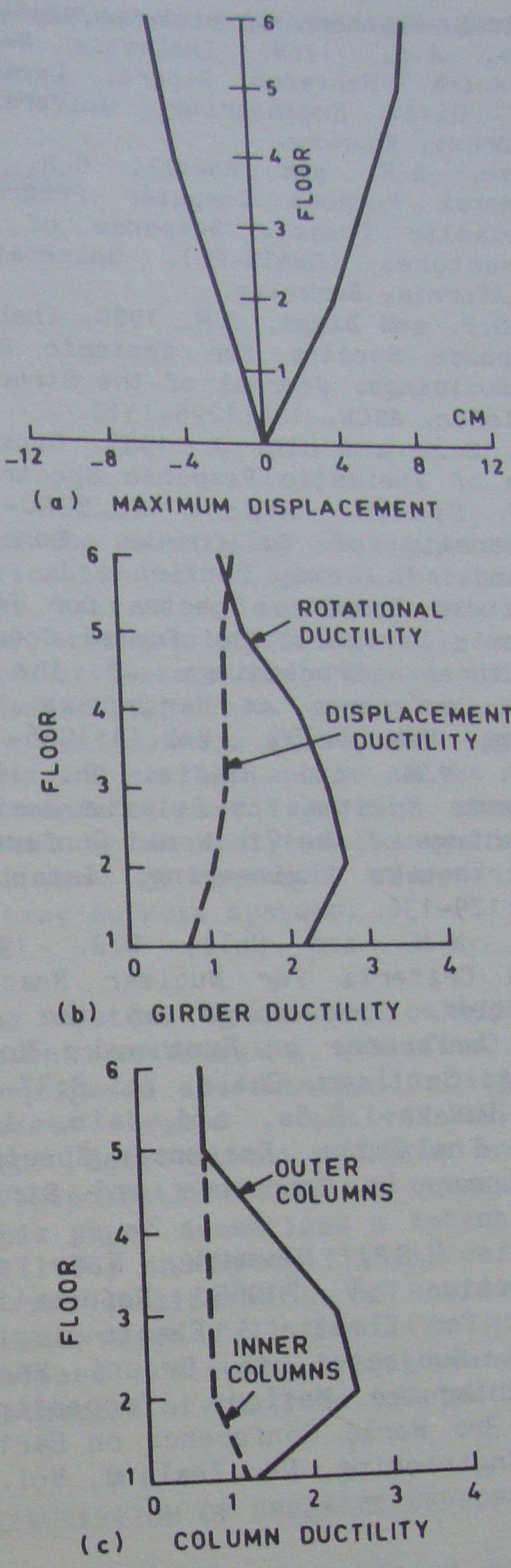

FIG. 10 RESPONSE OF FRAMES UNDER ARTIFICIAL EARTHQUAKE

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REFERENCES

Briseghella, L, Zaccuria, P.L. and Guffre, A. 1982. Inelastic Response Spectra. Proceedings of the 7th Symposium on Earthquake Engineering, University of Roorkee, Vol.1: 159-162.

Chandrasekaran, A.R. and Saini, S.S. 1967. Dynamic Response of Non-linear Systems. Journal of the Institution of Engineers,

Civil Engineering Division, 48:645-664.
Jain, A.K. 1985. Inelastic Response Spectra. Research Report, Department of Civil Engineering, University of Roorkee, Roorkee.
Kanaan, A.E. and Powell, G.H. 1973. General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures (DRAIN-2D). University of California, Berkeley.
Lai, S.P. and Biggs, J.M. 1980. Inelastic Response Spectra for Aseismic Design of Buildings. Journal of the Structural Division, ASCE, 106:1295-1310.
Mahin, S.A. and Lin, J. 1983. Construction of Inelastic Response Spectra for SDOF System. Report No. EERC-83/7, University of California, Berkeley.
Murakami, M. and Penzien, J. 1977. Nonlinear Response Spectra for Probabilistic Design of Reinforced Concrete Structures. Proceedings of the 6th World Conference on Earthquake Engineering, New Delhi, Vol. 11:1046-1051.
Newmark, N.M. and Riddle, R. 1980. Inelastic Spectra for Seismic Design. Proceedings of the 7th World Conference on Earthquake Engineering, Istanbul, Vol. 4:129-136.
Newmark, N.M. and Hall, W.J. 1969. Design Criteria for Nuclear Reactor Facilities. Proceedings of the 4th World Conference on Earthquake Engineering, Santiago, Chile, Vol. 2:37-50.
Pal, S., Dasaka, S.S., and Jain, A.K. 1987. Inelastic Response Spectra. (to appear in Computers and Structures).
Veletsos, A.S., Newmark, N.M. and Chelapati, C.V. 1965. Deformation Spectra for Elastic & Elasto-plastic Systems Subjected to Ground Shock and Earthquake Motions. Proceedings of the 3rd World Conference on Earthquake Engineering, New Zealand, Vol. 2: 663-680.